

# Faddeev calculation of a $K^-pp$ quasi-bound state

N.V. Shevchenko<sup>\*</sup>, <sup>1</sup> A. Gal,<sup>2</sup> and J. Mareš<sup>1</sup>

<sup>1</sup>*Nuclear Physics Institute, 25068 Řež, Czech Republic*

<sup>2</sup>*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*

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We report on the first genuinely three-body  $\bar{K}NN - \pi\Sigma N$  coupled-channel Faddeev calculation in search for quasi-bound states in the  $K^-pp$  system. The main absorptivity in the  $K^-p$  subsystem is accounted for by fitting to  $K^-p$  data near threshold. Our calculation yields one such quasi-bound state, with  $I = 1/2$ ,  $J^\pi = 0^-$ , bound in the range  $B \sim 55 - 70$  MeV, with a width of  $\Gamma \sim 90 - 110$  MeV. These results differ substantially from previous estimates, and are at odds with the  $K^-pp \rightarrow \Lambda p$  signal observed by the FINUDA collaboration.

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The issue of  $\bar{K}$  nuclear quasi-bound states has attracted considerable interest recently, motivated by earlier suggestions for (anti)kaon condensation in dense matter [1] and by extrapolations of  $K^-$  optical potentials fitted to  $K^-$  atom data [2, 3]. These  $K^-$  atom studies suggested  $\bar{K}$  nuclear potential depths about 150 – 200 MeV at nuclear-matter density  $\rho_0 = 0.16$  fm<sup>-3</sup>, although potentials evaluated by fitting to  $K^-p$  low-energy data give substantially lower values, about 100 MeV [4] or even as low as 50 MeV [5] depending on how the  $\bar{K}N$  in-medium  $t$  matrix is constructed. It was pointed out that  $\bar{K}$  nuclear states, if bound by over 100 MeV where the  $\bar{K}N \rightarrow \pi\Sigma$  main strong-decay channel is closed, might become sufficiently narrow to be observed [6, 7, 8]. Yamazaki and Akaishi [9], in particular, discussed few-body  $\bar{K}$  nuclear configurations in which the strongly attractive  $I = 0$   $\bar{K}N$  interaction is maximized. It is the  $I = 0$  coupled-channel  $s$ -wave interaction that generates a resonance in the  $\pi\Sigma$  coupled channel about 27 MeV below the  $K^-p$  threshold, the quasi-bound  $\Lambda(1405)$  [10]. The lightest  $\bar{K}$  nuclear configuration maximizing the  $I = 0$   $\bar{K}N$  interaction is the  $I = 1/2$   $\bar{K}(NN)_{I=1}$  state with  $S = L = 0$  and  $J^\pi = 0^-$  [11]. The significance of identifying this potentially low-lying quasi-bound state in the  $K^-pp$  mass spectrum of suitably chosen production reactions has been recently emphasized [12]. However, because the coupling of the two-body  $K^-p$  channel to the absorptive  $\pi Y$  channels was substituted by an energy-independent complex  $\bar{K}N$  potential, the results for binding energy and width of the  $K^-pp$  system [9] provide at best only a rough estimate. Recently, the FINUDA collaboration at DAΦNE, Frascati, presented evidence in  $K^-$  stopped reactions on several nuclear targets for the process  $K^-pp \rightarrow \Lambda p$ , interpreting the observed signal as due to a  $K^-pp$  deeply bound state [13]. However, this interpretation has been challenged in Refs. [3, 14]. Given this unsettled experimental search for a quasi-bound  $K^-pp$  state, precise three-body calculations for the  $K^-pp$  system appear well

motivated at present.

In this Letter we report on the first  $\bar{K}NN - \pi\Sigma N$  coupled-channel Faddeev calculation which is genuinely three-body calculation, searching for quasi-bound states that are experimentally accessible through a  $K^-pp$  final state. Coupled-channel three-body Faddeev calculations were reported for  $K^-d$ , with an emphasis on other entities than on quasi-bound states [15]. We note that the  $K^-d$  system is not as favorable as the  $K^-pp$  system for strong binding, since the relative weight of the  $I = 0$   $\bar{K}N$  interaction with respect to the weakly attractive  $I = 1$   $\bar{K}N$  interaction is 1 : 3 for  $K^-d$  and 3 : 1 for  $(K^-pp)_{I=1/2}$ . By doing coupled channel calculations, with two-body input fitted to available low-energy data, we wish to determine the scale of binding energy and width expected for few-body  $\bar{K}$  nuclear systems.

In the present work we solve non-relativistic three-body Faddeev equations in momentum space, using the Alt-Grassberger-Sandhas (AGS) form [16]. The AGS equations for three particles are:

$$\begin{aligned} U_{11} &= T_2 G_0 U_{21} + T_3 G_0 U_{31} \\ U_{21} &= G_0^{-1} + T_1 G_0 U_{11} + T_3 G_0 U_{31} \\ U_{31} &= G_0^{-1} + T_1 G_0 U_{11} + T_2 G_0 U_{21}, \end{aligned} \quad (1)$$

where  $G_0$  is the free three-body Green's function and  $T_i$ ,  $i = 1, 2, 3$ , are two-body T matrices in the three-body space for the pair excluding particle  $i$ . These equations define three unknown transition operators  $U_{ij}$  describing the elastic and re-arrangement processes:

$$\begin{aligned} U_{11} : & 1 + (23) \rightarrow 1 + (23) \\ U_{21} : & 1 + (23) \rightarrow 2 + (31) \\ U_{31} : & 1 + (23) \rightarrow 3 + (12), \end{aligned} \quad (2)$$

with Faddeev indices  $i, j = 1, 2, 3$  denoting simultaneously a given particle and its complementary interacting pair. Since the  $\bar{K}N$  two-body subsystem is strongly coupled to other channels, particularly via the  $\Lambda(1405)$  resonance to the  $I = 0$   $\pi\Sigma$  channel, it is necessary to extend the AGS formalism in order to include these channels explicitly. Thus, all operators entering the AGS equations

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\*Corresponding author: shevchenko@ujf.cas.cz

become  $3 \times 3$  matrices:  $G_0 \rightarrow G_0^{\alpha\beta} = \delta_{\alpha\beta} G_0^\alpha$  which is diagonal in the channel space, and  $T_i \rightarrow T_i^{\alpha\beta}$  where  $\alpha, \beta$  are channel indices as follows:

$$T_1 = \begin{pmatrix} T_1^{NN} & 0 & 0 \\ 0 & T_1^{\Sigma N} & 0 \\ 0 & 0 & T_1^{\Sigma N} \end{pmatrix} \quad (3)$$

$$T_2 = \begin{pmatrix} T_2^{KK} & 0 & T_2^{K\pi} \\ 0 & T_2^{\pi N} & 0 \\ T_2^{\pi K} & 0 & T_2^{\pi\pi} \end{pmatrix} \quad T_3 = \begin{pmatrix} T_3^{KK} & T_3^{K\pi} & 0 \\ T_3^{\pi K} & T_3^{\pi\pi} & 0 \\ 0 & 0 & T_3^{\pi N} \end{pmatrix}.$$

We assign particle labels (1, 2, 3) to  $(\bar{K}, N, N)$  in channel 1, to  $(\pi, \Sigma, N)$  in channel 2 and to  $(\pi, N, \Sigma)$  in channel 3. Here  $T^{NN}$ ,  $T^{\pi N}$  and  $T^{\Sigma N}$  are one-channel  $T$ -matrices, whereas  $T^{KK}$ ,  $T^{\pi\pi}$ ,  $T^{\pi K}$  and  $T^{K\pi}$  are the elements of the two-channel  $T^{KN-\pi\Sigma}$  matrix, accounting for  $\bar{K}N \rightarrow \bar{K}N$  and  $\pi\Sigma \rightarrow \pi\Sigma$  elastic processes, and for  $\bar{K}N \rightarrow \pi\Sigma$  and  $\pi\Sigma \rightarrow \bar{K}N$  inelastic transitions, respectively. We neglect the  $I = 1$  inelastic transition  $\bar{K}N \rightarrow \pi\Lambda$  since experimentally it is outweighed by the  $\bar{K}N \rightarrow \pi\Sigma$  transition, and also since the  $I = 1$   $\bar{K}N$  configuration plays a minor role in the structure of the  $I = 1/2 K^- pp$  system under discussion. Upon this extension into channel space, the unknown operators  $U$  assume the most general matrix form:  $U_{ij} \rightarrow U_{ij}^{\alpha\beta}$ . Substituting these new  $3 \times 3$  operators into the AGS system of equations we obtain the system to be solved.

Assuming charge independence, three-body quasi-bound states are labelled by isospin. The isospin basis is used throughout our calculation within a coupling scheme that ensures that we are searching for an  $I = 1/2$  quasi-bound state. Assuming pairwise  $s$ -wave meson-baryon interactions, as appropriate to the  $\bar{K}N - \pi\Sigma$  system near the  $\bar{K}NN$  threshold, and  $s$ -wave baryon-baryon interactions limited to the  ${}^1S_0$  configuration as appropriate to  $pp$ , the total spin and total orbital angular momentum of the three-body system are  $S = L = 0$ . Tensor forces are not operative for this situation, which also reinforces the neglect of coupling to  $\pi\Lambda N$  since the strong  $\Sigma N \rightarrow \Lambda N$  transition is dominated by the tensor force in the  ${}^3S_1 YN$  configuration. Hence, the  $\bar{K}NN - \pi\Sigma N$  system explored in this Faddeev calculation has quantum numbers  $I = 1/2$ ,  $L = 0$ ,  $S = 0$ ,  $J^\pi = 0^-$ .

In order to reduce the dimension of the integral equations, a separable approximation for the two-body  $T$  matrices is used:

$$T_{i,I_i}^{\alpha\beta} = |g_{i,I_i}^\alpha\rangle \tau_{i,I_i}^{\alpha\beta} \langle g_{i,I_i}^\beta|, \quad (4)$$

where  $I_i$  is the conserved isospin of the interacting pair. [for  $\alpha = \beta$  our generalized  $T$ -matrices coincide with the usual ones.] For separable two-body  $T$ -matrices, the AGS equations may be rewritten using a new kernel and unknown functions:

$$Z_{ij,I_i I_j}^{\alpha\beta} \equiv \delta_{\alpha\beta} \langle g_{i,I_i}^\alpha | G_0^\alpha | g_{j,I_j}^\beta \rangle \quad (5)$$

$$X_{ij,I_i I_j}^{\alpha\beta} \equiv \langle g_{i,I_i}^\alpha | G_0^\alpha U_{ij,I_i I_j}^{\alpha\beta} G_0^\beta | g_{j,I_j}^\beta \rangle, \quad (6)$$

TABLE I: Strength parameters  $\lambda_{I=0,1}^{\alpha\beta}$  (in units  $\text{fm}^{-2}$ ) for the  $\bar{K}N - \pi\Sigma$  potentials (7) with range parameter  $\beta = 3.5 \text{ fm}^{-1}$ , corresponding to  $a_{K-p} = (-0.70 + i 0.60) \text{ fm}$ .

$\lambda_{I=0}^{\bar{K}N, \bar{K}N}$	$\lambda_{I=0}^{\bar{K}N, \pi\Sigma}$	$\lambda_{I=0}^{\pi\Sigma, \pi\Sigma}$	$\lambda_{I=1}^{\bar{K}N, \bar{K}N}$	$\lambda_{I=1}^{\bar{K}N, \pi\Sigma}$	$\lambda_{I=1}^{\pi\Sigma, \pi\Sigma}$
-1.370	1.414	-0.176	0.007	1.734	-0.340

respectively. The calculation of the kernels  $Z$  involves transformation from one set of Jacobi coordinates to another one and isospin recoupling as well. The position of the three-body pole was searched as a zero of the determinant of the kernel of the system of integral equations on the corresponding unphysical sheet. More details on the extended AGS equations and the numerical procedure are relegated to an expanded version of this paper. Here it suffices to mention that by assuming charge independence,  $s$ -wave pairwise interactions, and antisymmetrizing over the two nucleons, we end up in a system of nine coupled integral equations. This is the *minimal* dimensionality of any Faddeev calculation in the  $I = 1/2 J^\pi = 0^-$  sector which attempts to account explicitly for the strong absorptivity of the  $\bar{K}N$  interactions near threshold. Within this scheme, the interaction of the relatively energetic pion with the slow baryons was neglected, partly because its  $p$ -wave nature would require an extension of the present  $s$ -wave calculation.

The input separable potentials for the  $T$ -matrices (4) are given in momentum space by

$$V_I^{\alpha\beta}(k_\alpha, k'_\beta) = \lambda_I^{\alpha\beta} g_I^\alpha(k_\alpha) g_I^\beta(k'_\beta), \quad (7)$$

where  $k_\alpha, k'_\beta$  are two-particle relative momenta in the two-body respective channels, and  $\lambda_I^{\alpha\beta}$  are strength-parameter constants. For the  $\alpha = \beta = (NN)_{I=1}$  channel, we have used a separable approximation of the Paris potential [17], corresponding to the one-rank potential (7) with  $\lambda_I^{NN} = -1$  and a form factor:

$$g_{I=1}^{NN}(k) = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^6 \frac{c_{i,I=1}^{NN}}{k^2 + (\beta_{i,I=1}^{NN})^2}. \quad (8)$$

The constants  $c_{i,I=1}^{NN}$  and  $\beta_{i,I=1}^{NN}$  are listed in Ref. [17].

For the  $S = -1$  interactions, the form factors  $g_I^\alpha(k_\alpha)$  in Eq. (7) were parameterized by a Yamaguchi form

$$g_I^\alpha(k^\alpha) = \frac{1}{(k^\alpha)^2 + (\beta_I^\alpha)^2}. \quad (9)$$

For the  $I = 3/2 \Sigma N$  interaction we made two different choices of  $\lambda_{I=3/2}^{\Sigma N}$  and  $\beta_{I=3/2}^{\Sigma N}$ . The first choice, labelled (i) below, reproduces the scattering length  $a_{I=3/2} = 3.8 \text{ fm}$  and effective range  $r_{I=3/2} = 4.0 \text{ fm}$  of the Nijmegen Model F [18]. The second choice, labelled (ii) below, reproduces the most recent Nijmegen  $YN$  phase shifts [19] using a scattering length  $a_{I=3/2} = 4.15 \text{ fm}$  and effective range  $r_{I=3/2} = 2.4 \text{ fm}$ . For the  $I = 1/2 \Sigma N$  interaction

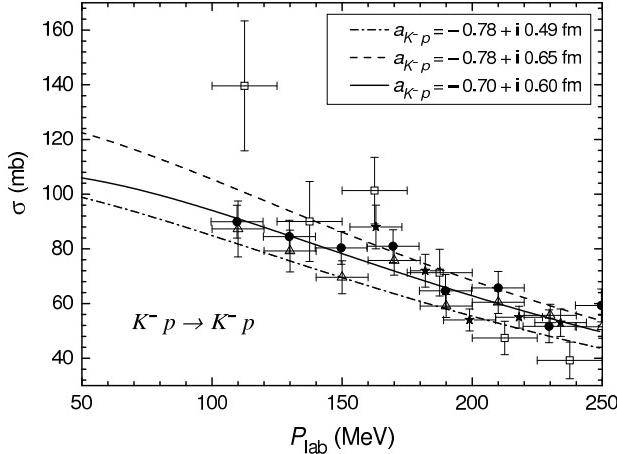


FIG. 1: Calculated  $K^- p \rightarrow K^- p$  cross sections, for three different sets of  $\bar{K}N - \pi\Sigma$  parameters, in comparison with the measured cross sections (see text).

we reproduced the value quoted by Dalitz [20] for the scattering length  $a_{I=1/2} = -0.5$  fm.

For the  $I = 0, 1$   $\bar{K}N - \pi\Sigma$  coupled-channel potentials, the parameters  $\lambda_{I=0,1}^{\alpha\beta}$  and  $\beta_{I=0,1}^\alpha$  in Eqs. (7, 9) were fitted to reproduce (i)  $E_{\Lambda(1405)} = 1406.5 - i 25$  MeV [10], the position and width of  $\Lambda(1405)$  which is assumed to be a quasi-bound state in the  $\bar{K}N$  channel and a resonance in the  $\pi\Sigma$  channel, (ii) the branching ratio at rest [21]  $\gamma = \Gamma(K^- p \rightarrow \pi^+ \Sigma^-)/\Gamma(K^- p \rightarrow \pi^- \Sigma^+) = 2.36$ , and (iii) the  $K^- p$  scattering length  $a_{K^- p}$  for which we used as a guideline the KEK measured value [22]:

$$a_{K^- p} = (-0.78 \pm 0.15 \pm 0.03) + i(0.49 \pm 0.25 \pm 0.12) \text{ fm}. \quad (10)$$

In order to check the sensitivity of our results to this input, within the quoted errors, we fitted three different values of  $a_{K^- p}$  using a range parameter  $\beta = 3.5 \text{ fm}^{-1}$ . All three sets of our  $\bar{K}N - \pi\Sigma$  parameters, which also reproduce the energy and width of  $\Lambda(1405)$  and the branching-ratio  $\gamma$ , yield low-energy  $K^- p \rightarrow K^- p$  and  $K^- p \rightarrow \pi^+ \Sigma^-$  cross-sections which are in a good agreement with experimental data, as shown in Figs. 1 and 2. We note that the data points in these figures are precisely those compiled and cited in Ref. [24]. The strength parameters  $\lambda_{I=0,1}^{\alpha\beta}$  for the  $\bar{K}N - \pi\Sigma$  coupled-channel separable potentials fitted to  $a_{K^- p} = (-0.70 + i 0.60) \text{ fm}$  are given for illustration in Table I.

In a test calculation we first switched off the coupling of the  $\bar{K}NN$  channels to the  $\pi\Sigma N$  channels. This reduces the number of coupled integral equations from nine to three within the three-body  $\bar{K}NN$  space. We assumed the  $\Lambda(1405)$  to be a genuine bound state of the  $(\bar{K}N)_{I=0}$  subsystem, reproducing the real part of the  $K^- p$  scattering length of Eq. (10). We found a zero-width bound state at energy  $E_{\bar{K}NN} = -43.7$  MeV below the  $\bar{K}NN$  threshold. This binding energy is considerably larger than the value  $E_{\bar{K}NN} \approx -10$  MeV estimated by

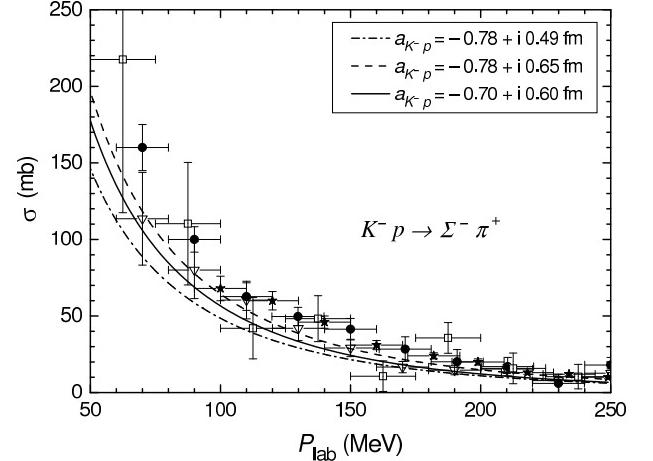


FIG. 2: Calculated  $K^- p \rightarrow \pi^+ \Sigma^-$  cross sections, for three different sets of  $\bar{K}N - \pi\Sigma$  parameters, in comparison with the measured cross sections (see text).

Nogami [11]. We then performed full  $\bar{K}NN - \pi\Sigma N$  three-body calculations for the three sets of  $\bar{K}N - \pi\Sigma$  parameters and for the two sets (i) and (ii) of  $(\Sigma N)_{I=3/2}$  parameters described above. The sensitivity of the results to the  $\Sigma N$  interaction was also studied by setting  $T^{\Sigma N} = 0$  for both  $I = 1/2$  and  $I = 3/2$ . The calculated binding energies ( $B = -E_B$ ) and widths ( $\Gamma$ ) are presented in Table II where the energies are given with respect to the  $K^- pp$  threshold. It is seen that the  $\Sigma N$  interaction, dominantly in the  $I = 3/2$  channel, adds only about 3 MeV to the binding energy (less than 6%) affecting the width by up to 2 MeV (less than 2%). This is negligible on the scale of binding energies and widths displayed in the table and is consistent with the negligible effect (less than 2%) that the  $YN$  and  $\pi N$  final-state interactions were found to have in the latest  $K^- d$  Faddeev calculation of Ref. [15]. In contrast, the calculated binding energies and widths show sensitivity to the fitted  $\bar{K}N - \pi\Sigma$  coupled-channel two-body interactions, giving rise in our calculations to up to about 25% variation in  $B$  and up to about 15% variation in  $\Gamma$ . It is worth noting that  $B$  increases with  $\text{Im } a_{K^- p}$ , whereas  $\Gamma$  is correlated more with  $\text{Re } a_{K^- p}$ ; this feature is typical to strong-absorption phenomena where the width gets saturated beyond a critical value of absorptivity [25]. We have also studied the dependence of the calculated binding energy and width on the range parameter  $\beta$  within acceptable fits, keeping  $a_{K^- p}$  constant, say  $a_{K^- p} = (-0.78 + i 0.49) \text{ fm}$ . The binding energy changes very little, by about 3 MeV, whereas the width changes appreciably, decreasing from 115 MeV for  $\beta = 3 \text{ fm}^{-1}$  to 89 MeV for  $\beta = 4 \text{ fm}^{-1}$ .

Our calculations confirm the existence of an  $I = 1/2$ ,  $J^\pi = 0^-$  three-body quasi-bound state, with appreciable width, in the  $\bar{K}(NN)_{I=1}$  channel. The width of this quasi-bound state is a measure of its coupling to the  $\pi\Sigma N$  channels where it shows up as a broad resonance. The coupling to the  $\pi\Sigma N$  channels, in addition to providing a width which renders the  $\bar{K}(NN)_{I=1}$  bound state into

TABLE II: Calculated energy  $E_{\bar{K}NN} = E_B - i \Gamma/2$  (in MeV) of the  $I = 1/2$ ,  $J^\pi = 0^-$  quasi-bound  $\bar{K}(NN)_{I=1}$  state with respect to the  $K^-pp$  threshold, calculated for different two-body input.  $E^{(i)}$  and  $E^{(ii)}$  correspond to sets (i) and (ii), respectively, of the  $(\Sigma N)_{I=3/2}$  interaction parameters, whereas  $E^{(0)}$  stands for no  $\Sigma N$  interaction (see text).

$a_{K^-p}$ (fm)	$E_{\bar{K}NN}^{(i)}$ (MeV)	$E_{\bar{K}NN}^{(ii)}$ (MeV)	$E_{\bar{K}NN}^{(0)}$ (MeV)
$-0.78 + i 0.49$	$-55.8 - i 49.1$	$-56.2 - i 50.1$	$-53.4 - i 49.2$
$-0.78 + i 0.65$	$-69.4 - i 46.8$	$-70.0 - i 47.9$	$-66.3 - i 47.5$
$-0.70 + i 0.60$	$-66.0 - i 54.7$	$-66.5 - i 55.8$	$-63.5 - i 54.6$

a quasi-bound state, also provides substantial extra attraction through which the binding energy is increased from 44 MeV to the range of values shown in the table. The acceptable parameter sets considered in our calculations yield binding in the range  $B \sim 55 - 70$  MeV, with a width of  $\Gamma \sim 90 - 110$  MeV. Although the binding energy calculated here is similar to that estimated by Yamazaki and Akaishi [9] for  $K^-pp$ , our calculated width is considerably larger than their estimate  $\Gamma = 61$  MeV and is also larger than the width  $\Gamma \approx 67$  MeV of the  $K^-pp \rightarrow \Lambda p$  signal in the FINUDA experiment [13]. Our range of calculated binding energies is considerably lower than  $B \approx 115$  MeV attributed by the FINUDA collab-

oration to a  $K^-pp$  bound state. Possible extensions of the present coupled-channel Faddeev calculation should include the  $I = 1 \pi\Lambda$  channel, enlarge the model space to include  $p$ -wave two-body interactions and introduce relativistic kinematics. Relying on the experience of coupled-channel Faddeev calculations of the  $K^-d$  system [15], none of these extensions is expected to change *qualitatively* our results and conclusions.

In conclusion, we performed the first coupled-channel three-body Faddeev calculation for the  $I = 1/2$   $\bar{K}(NN)_{I=1}$  system in search of a quasi-bound state. This state can be reached in production reactions aiming at a final  $K^-pp$  system. It is primarily the large width, here calculated for a  $\bar{K}$  nuclear state above the  $\pi\Sigma$  two-body threshold, that poses a major obstacle to observing and identifying  $\bar{K}$  nuclear quasi-bound states. Yet, even for deeper states below the  $\pi\Sigma$  threshold, in heavier nuclei, a residual width of order 50 MeV is expected to persist due to  $\bar{K}NN \rightarrow YN$  absorption [3].

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